WEEK 11: GEOMETRY – TRIANGLES

Triangle Inside a Rectangle

A triangle has two shared vertices and one shared side with a rectangle. The third vertex is anywhere on the side opposite of the shared side.

How does the area of the triangle compare with the area of the rectangle? Why do you think this relationship holds?

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<tr>
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<tbody>
<tr>
<td>M6G1. Students will further develop their understanding of plane figures.</td>
<td>Students can use GSP to construct the triangle in a rectangle, find the areas of both and compare. Then students can manipulate the triangle to see if conjecture holds.</td>
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<td>M6A1. Students will understand the meaning of ratio and how to use it.</td>
<td>The area of the triangle to the area of the rectangle is in a ratio of 1:2. The students set up the area formulas as ratios: Area of the Triangle/Area of Rectangle = (1/2bh)/bh = 1/2.</td>
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<td>M7G1. Students will construct plane figures that meet given conditions</td>
<td>Students can use GSP to construct the triangle in a rectangle, and can manipulate the third vertex to devise and test their conjecture.</td>
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<td>M8G1. Students will analyze and use characteristics and properties of geometric figures. a.) Use the properties of parallel and perpendicular lines. c.) Use and apply the properties of triangles and parallelograms.</td>
<td>Students can show congruent triangles by dropping a perpendicular line from the third vertex to the base. To show congruent triangles, extend the sides of the rectangles as lines. These are parallel to the perpendicular line.</td>
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**Triangle Inside a Rectangle, con’t**

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| **M8G2.** Students will use the properties of similarity and congruency and apply these concepts to geometric figures.  
  b. Understand the properties of the ratio of segments of parallel lines cut by one or more transversals.  
  c. Understand the meaning of congruency and the conditions for congruent triangles and other polygons.  
  d. Use properties to determine similarity and congruency of triangles. | Students can show congruent triangles by dropping a perpendicular line from the third vertex to the base. To show congruent triangles, extend the sides of the rectangles as lines. These are parallel to the perpendicular line. |
| **M8A1.** Students will represent, solve, and analyze mathematical situations algebraically.  
  a.) Simplify and evaluate algebraic expressions. | Students can use the formulas for area of a triangle and the area of a rectangle, and because the height and base are the same value, the student can simplify to determine a relationship. |
**Half as much may be right**

What is significant about an inscribed (blue) angle in a semi-circle? Why might this fact be true?

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<td><strong>M6G1.</strong> Students will further develop their understanding of plane figures.</td>
<td>Students can use GSP to construct the triangle in the semicircle and then measure the degree of the angle. The students will develop a better understanding of inscribed angles.</td>
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| **M7G1.** Students will construct plane figures that meet given conditions. They will also demonstrate understanding of transformations.  
  a. Make basic constructions using a compass and straight edge. | Students will construct the semi-circle and inscribed angle either with a compass and straight edge or GSP. Students can then measure the angle. If using GSP, the students can dynamically move the point along the semi-circle. If using a compass and straight-edge, the student can position the angle in various locations on the semi-circle and measure the angle. |
| **M7A1** Students will represent and evaluate quantities using algebraic expressions.  
  b. Use and evaluate algebraic expressions  
  c. Add and subtract linear expressions. | In solving this problem algebraically, the students can divide the inscribed triangle into two separate triangles. Let the angles for the two triangles be as follows: <1, <2, <3 for one triangle, and <4, <5, <6 for the other with (<1 + <4) being equal to the inscribed angle (blue). Using various properties of triangles (see M8G1) and supplementary angles, the following algebraic equations can be set up for the angle sum of the triangles:  
  <1 + <2 + <3 = 180  
  <4 + <5 + <6 = 180  
  Since we are dealing with isosceles triangles, the base angles are congruent (so we can substitute one for another in the above equations)  
  2(<1) + <3 = 180  
  2(<4) + <6 = 180  
  Adding the equations together and using the fact that supplementary angles add to 180, we eventually get <1+<4 = 90 |
**Half as much may be right, con’t**

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| **M8G1.** Students will analyze and use characteristics and properties of geometric figures.  
b.) Use and apply properties of angle pairs such as complementary, supplementary and vertical angles.  
c.) Use and apply the properties of triangles and parallelograms. | To solve this problem algebraically, the property of supplementary angles is needed and the following properties of a triangle are needed: the angle sum of a triangle is 180 degrees, and what constitutes an isosceles triangle, and that base angles of an isosceles triangle are congruent. |
| **M8A1** Students will represent, solve, and analyze mathematical situations algebraically.  
a.) Simplify and evaluate algebraic expressions.  
c.) Solve algebraic expressions. | See M7A1. |
| **M8A6** Students will use and simplify monomials and polynomials.  
a.) Add and subtract simple monomials and polynomials | See M7A1. |